A MILP optimization model for the dock-assignment in a multi-door cross-docking hub

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Abstract: Cross-docking is a logistic strategy for shipping goods from suppliers to customers without intermediate storage, using a hub to exchange freights from one truck to another. Goods are collected from several suppliers via inbound trucks, unloaded in the cross-dock hub, reconsolidated with other goods and loaded into outbound trucks heading toward the customers. This study focuses on optimising the handling activities required to unload, move, and reload the goods inside cross-dock terminals by developing and applying an original mixed-integer linear programming model. This model aims to determine the optimal solution for formulating the dock-assignment problem in a multi-door cross-docking system, where multiple trucks can simultaneously unload (inbound) and reload (outbound) palletised until-loads. The objective function minimises the travelling time spent by forklifts moving palletised until-loads from inbound to outbound docks. Furthermore, a twin hub nearby provides extra doors for cross-docking operations. The model identifies and controls the trade-off between truck-based and shuttle-based transferring activities among hubs. The proposed model is applied to a cross-docking hub made of two twin facilities owned by an Italian company specialised in the fast delivery of palletised unit-loads (called loads). A comparative what-if multi-scenario analysis demonstrates the effectiveness of the proposed model in reducing the time-to-shipping of loads.

Keywords: cross-docking, truck dock assignment, multi-door and multi-truck cross-dock, optimisation model, mixed-integer linear programming (MILP)

I. INTRODUCTION

Cross-docks are logistic terminals where goods are unloaded from inbound trucks, sorted and moved from one door to another—within the cross-dock—until they are finally loaded on outbound trucks towards the next destination within the distribution network [1]. Therefore, goods can be temporally stored only for a short time [2]. This warehouse solution aims to reduce the shipping time and cost by reducing (or removing) storage and order picking activities, thus accelerating the flow of the shipping cycle [3].

The growing trend of just-in-time and continuous flow management in supply chains has brought the blooming of cross-docking warehouses to face the increasing goods flow and the customers' need to shorten delivery lead times. Therefore, modern distribution systems require increasingly large hubs. Space is a limited resource in many contexts.

Multi-door cross-docking hubs are widespread primarily to facilitate material handling activities by minimising indoor congestion and waiting times. Thus, new twin hubs are built nearby the main ones, i.e., the first built hub. On the one hand, a twin hub increases the storage capacity and reduces waiting times and indoor congestion. On the other hand, it generates new material flows between hubs (called inter-hubs flows) that must be controlled and managed. Most of the literature pays attention to layout design [4], distribution network, vehicle routing [5], door assignment [6-8] and truck scheduling [9-11] problems, focusing on a single cross-docking hub. This study exploits the dock assignment problem in the presence of a twin hub by the application to a real-world case study. In particular, an original mixed-integer linear programming (MILP) model is presented. The aim is to solve the hub and dock assignment problems assuming two twin multi-door cross-docking hubs located nearby, where multiple trucks can simultaneously unload and reload palletised until-loads (called loads).

The remainder of this paper is organised as follows: Section 2 illustrates a literature review concerning cross-dock. Section 3 describes the proposed MILP model, and Section 4 presents an application of this model to a real case study. Finally, Section 5 shows the conclusions and outlines for future research.

II. LITERATURE REVIEW

Studies about the cross-docking technique are relatively recent. The first contributions are accountable to Tsui and Chang [12-13], who illustrated a tool and a bilinear programming model to solve the dock-door assignment problem. Many researchers have paid attention to cross-dock hub management in the last decades. The most related studies to this paper were presented by Oh et al.

[13], Miao et al. [15], Liao et al. [16], Kucukoglu and Ozturk [7]. These papers focused on the truck-door assignment problem by optimising several performance indicators, such as tardiness, makespan and travel time. In particular, Assadi and Bagheri [11] presented the simplifying assumptions that limit the real-world applicability in most studies on the cross-dock hub. They illustrated three limiting assumptions:

1. The cross-dock has just one receiving and one shipping door.
2. All trucks are considered available at the start of the planning horizon.
3. The travel times measured from receiving doors to shipping doors are considered equal (i.e., they don’t consider the real distance between doors)

This paper aims to fill this gap by overcoming implications (1, 3) and answering a real-application implication. To this, a twin hub configuration is introduced. Assumption (2) is ignored because it is out of the scope of this study. Therefore, the proposed MILP model supports the planning and management activities that involve the two twin hubs by answering three main questions:

1. In which hub does the truck unload loads?
2. In which door?
3. How are the loads’ inter-hubs flows managed?

Finally, this model is applied to a case study to compare the proposed solutions with the real company policy.

III. Problem formulation

Cross-docks are logistic terminals where loads retrieved by inbound trucks are sorted and carried out to shipping doors upon customer order. These handling operations involve three steps: (1) the unloading of the loads from the inbound truck (receiving), (2) the handling until the shipping door (sorting), and (3) the loading on the outbound truck (shipping). Figure 1 presents the three steps that identify a “standard cross-dock”.

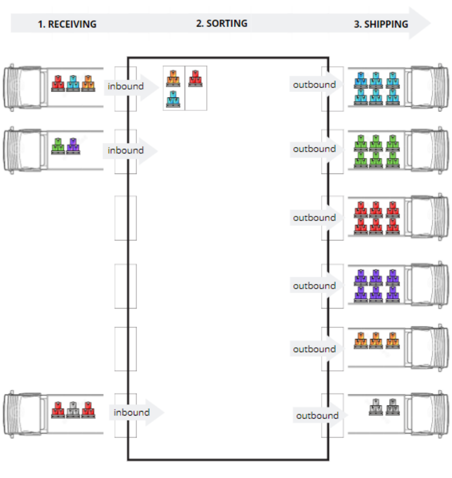

Fig. 1. Standard cross-dock

In this case, the *door assignment* is the only variable influencing handling time.

The presence of the twin hub allows splitting the loads inbound/outbound flow into the two hubs, increasing the storage capacity by duplicating the inbound/outbound doors. Therefore, a new variable is introduced: the *hub assignment*. Finally, a third variable is introduced to manage the flow of the loads unloaded in the hub chosen by the truck and destined to the other hub for the sorting and shipping activities: we define it as *the inter-hubs load flow management*. This flow can be handled in two ways:

1. By “shuttle” (also called SH). Loads unloaded in one of the two hubs (e.g., door 1 in Fig. 2) are loaded on the shuttle and transported to the twin hub. The departure door of the shuttle (e.g., door 2 in Fig. 2) is fixed per hub, while the receiving door (e.g., door 3 in Fig. 2) is arbitrarily chosen to reduce the distance travelled by the loads to the shipping doors (e.g., doors 4 in Fig. 2). Figure 2 shows the main steps.

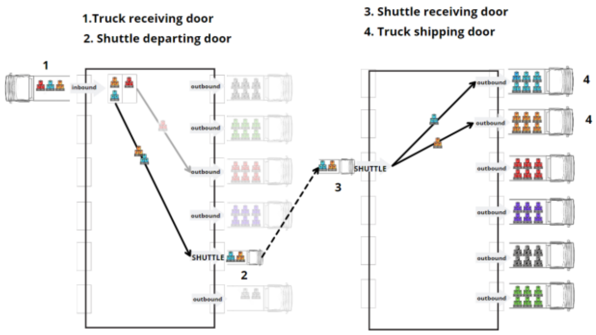

Fig. 2. Cross-dock standard and shuttle

2. By “truck double handling” (also called TDH). Loads for the twin hub are re-loaded onto the truck at the receiving door (e.g., door 1 in Fig. 3) and transported to the hub. The assumption is that the receiving door (e.g., door 2 in Fig. 3) chosen for the second truck unload is the one where that truck will fill up for the shipping. Loads are finally moved to the shipping doors (e.g., doors 3 in Fig. 3). This rule is inspired by the case study TDH policy (illustrated in Section 5) to best design the real-applicability implications of a cross-dock hub. Figure 3 shows the main steps.

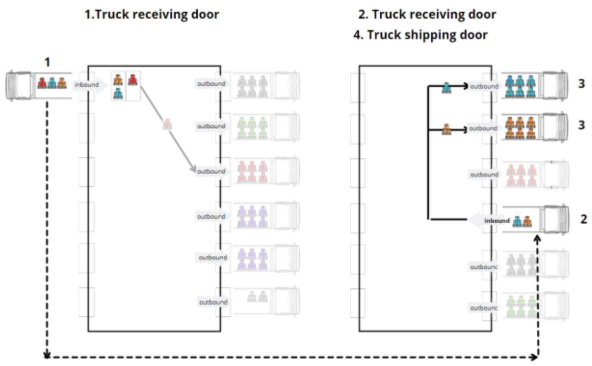

Fig. 3. Cross-dock standard and truck double handling (TDH)

Therefore, this study aims to manage all the scenarios: (1) the standard cross-dock, (2) the standard cross-dock joint to the handling by shuttle, and (3) the standard cross-dock joint to the truck double handling (TDH). Consequently, given a generic inbound truck or a set of inbound trucks, three interrelated problems need to be faced:

1. hub-assignment problem
2. door-assignment problem
3. inter-hubs load flows management problem

The problems mentioned above are modelled through a MILP model to minimise the travelling time spent moving loads from inbound to outbound doors. The model notation and formulation are following.

TABLE I
MILP PROBLEM: SETS

Set	Description
$h = 1, 2$	*number of hubs*
$m = 1, \dots M$	*number of inbound trucks*
$s = 1, \dots S$	*receiving doors*
$i = 1, \dots I$	*receiving doors in hub 1 ($I \subset S$)*
$f = 1, \dots F$	*receiving doors in hub 2 ($F \subset S$)*
$c = 1, \dots C$	*shipping doors*
$j = 1, \dots J$	*shipping doors in hub 1 ($J \subset C$)*
$p = 1, \dots P$	*shipping doors in hub 2 ($P \subset C$)*
$n = n_1, n_2$	*departure door for the shuttle in hub 1 ($n_1 \subset N$) and hub 2 ($n_2 \subset N$)*
$ds = 1, \dots, DS$	*departure doors for TDH*
$ds_1 = 1, \dots, DS1$	*receiving doors for TDH in hub 1 ($DS1 \subset DS$)*
$ds_2 = 1, \dots, DS2$	*receiving doors for TDH in hub 2 ($DS2 \subset DS$)*

TABLE II
MILP PROBLEM: PARAMETERS

Parameter	Description
$DIST_{s,c}$	*matrix of distances between door pairs (s;c) [m]*
V	*speed of the internal handling vehicle [m/s]*
A	*acceleration (with load) of the internal handling vehicle [m²/s]*
A_e	*acceleration (without load) of the internal handling vehicle [m²/s]*
$TIME_{sc}$	*matrix of travelling times between door pairs (s;c) considering vehicle speed and accelerations [s]*
P_{mh}	*number of loads on truck m that must be shipped from hub h [load]*
PD_{mc}	*number of loads on truck m direct towards door c*
T	*handling time for moving loads to the departure door of the shuttle [s]*
T_{mov}	*truck handling time for TDH [s]*
T_{op}	*time to arrange the shuttle [s]*
T_SH	*time for loading/unloading the load on/off the shuttle [s]*
T_TDH	*time for loading/unloading the load on/off the truck in the TDH [s]*
W	*shuttle capacity [load/trip]*
RD_{mds}	*assignment matrix "truck m"–"charging door ds"*

Variables

A set of binary and integer variables are now introduced to manage the hub-door assignment problems.

$$q_{mh} = \begin{cases} 1 & \text{if the truck m unloads in the Hub h} \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

where $q_{mh} \in \{0; 1\}$, $m \in M$, and $h \in H$

$$y_{ms} = \begin{cases} 1 & \text{if the truck m unloads at door s} \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

where $y_{ms} \in \{0; 1\}$, $m \in M$, and $s \in S$

Variables (1) and (2) ensure that each truck unloads its loads in one hub (1), at one door (2). Moreover, if the truck transports loads that must be consolidated and shipped from the twin hub, variables (3) and (4) guarantee that only one door is chosen respectively for the receiving of the shuttle and for the departure of the truck.

$$ynav_{ms} = \begin{cases} 1 & \text{if the loads of truck m are unloaded at the door s by the shuttle} \\ 0 & \text{otherwise} \end{cases} \tag{3}$$

$$ytruck_{ms} = \begin{cases} 1 & \text{if the door s is chosen as starting point by the truck m before going to the twin hub} \\ 0 & \text{otherwise} \end{cases} \tag{4}$$

where $ynav_{ms}, ytruck_{ms} \in \{0; 1\}$, $m \in M$, and $s \in S$

In addition, some integer variables are introduced to quantify the intra- and inter-hubs load flows. Firstly, variable (5) quantifies the travelling time required to move loads according to a standard cross-dock. Therefore, all flows starting from a receiving door s, directed towards a shipping door c, are considered.

$$x_{msc} \geq 0 \quad \text{where } m \in M, s \in S, c \in C \tag{5}$$

Moreover, variables (6) and (7) illustrate respectively the loads flow from hub 1 (h=1) to hub 2 (h=2) and vice versa. Both are managed by the previously introduced shuttle. Therefore, all load flows starting from a receiving door i in hub 1 (or f in hub 2), which are moved by shuttle from the departure door n_1 (n_2) to the receiving door f (i), and which are finally led up to the shipping doors p (j), are considered.

$$xnav_{mhinfp} \geq 0 \tag{6}$$

where $m \in M$, $h = 1$, $i \in I$, $n = n_1$, $f \in F$, $p \in P$

$$nnav_{mhfnij} \geq 0 \tag{7}$$

where m ∈ M, h = 2, f ∈ F, n = n_2, i ∈ I, j ∈ J

Finally, variables (8) and (9) illustrate respectively the loads flow from hub 2 (*h=2)* to hub 1 *(h=1)* and vice versa, managed according to the TDH option. In particular, all loads flow starting from a receiving door i *(f)* in hub 1, which are reloaded into the trucks and moved to the other hub until the door ds_1(ds_2), which are finally led up to the shipping doors *p (j),* are considered.

$$xtruck_{mhidsp} \geq 0 \quad (8)$$

where m ∈ M, h = 1, i ∈ I, ds = ds_2, p ∈ P

$$ntruck_{mhfdsj} \geq 0 \quad (9)$$

where m ∈ M, h = 2, f ∈ F, ds = ds_1, j ∈ J

Objective function

This is the proposed MILP object function:

min

$$\sum_m \sum_i \sum_j x_{mij}\, TIME_{ij} + \sum_m \sum_f \sum_p x_{mfp}\, TIME_{fp} + \sum_m \sum_{h=1} \sum_i \sum_{n\in n1} \sum_f \sum_p xnav_{mhinfp}\, TIME_{fp} + \sum_m \sum_{h=2} \sum_f ynav_{mf}\left(T\_SH\, P_{m,h} + T_{op}\right) + \sum_m \sum_{h=2} \sum_f \sum_{n\in n2} \sum_i \sum_j nnav_{mhfnij}\, TIME_{ij} + \sum_m \sum_{h=1} \sum_i ynav_{mi}\left(T\_SH\, P_{m,h} + T_{op}\right) + \sum_m \sum_{h=1} \sum_i \sum_{ds\in ds2} \sum_p xtruck_{mhidsp}\, TIME_{dsp} + \sum_m \sum_{h=2} \sum_i ytruck_{m,i}\left(T_{mov} + T\_TDH\, P_{mh}\right) + \sum_m \sum_{h=2} \sum_f \sum_{ds\in ds1} \sum_j ntruck_{mhfdsj}\, TIME_{dsj} + \sum_m \sum_{h=1} \sum_f ytruck_{m,f}\left(T_{mov} + T\_TDH\, P_{mh}\right) \quad (10)$$

It is made of ten different time contributions. The aim is to minimise the total time required to retrieve, sort and ship the loads. Both standard cross-dock and inter-hubs flows (shuttle-based and truck-based) are considered.

The first two contributions of equation 10 represent the time needed to unload, move, and reload loads according to standard cross-dock in hub 1 and hub 2, respectively.

The third and fourth contributions measure the time required to manage the loads by a standard cross-dock with an additional handling task by shuttle from hub 1 to hub 2. In particular, the third contribution illustrates the time required to move the loads from the receiving door of the shuttle *f* to the shipping door *p* for each load handled in hub 2. The fourth contribution measures the time required to move the loads from the receiving door *i* in hub 1 to the departure door of the shuttle (n_1). The time needed for the operator to arrange the shuttle is shown by T_{op}, while the time to load/unload each load is presented by T_SH. Finally, the average waiting time for the shuttle is introduced by parameter T. Similarly, the fifth and sixth contributions consider the shuttling from hub 2 to hub 1.

Finally, the seventh and eighth contributions measure the time necessary to handle loads by a standard cross-dock with an additional handling task by TDH. In particular, the seventh contribution measures the time to move loads from the receiving door *ds_2* of each truck *m* to the shipping door *p* for each load handled in hub 2. The eighth contribution represents the time required to reload loads in hub 1, considering a fixed loading/unloading time of T_TDH per load and a manoeuvring time quantified by T_{mov}. Similarly, the ninth and the tenth contributions consider the TDH in hub 1.

Constraints

The set of constraints is illustrated as follows. The generic truck must choose the "receiving hub" (11) and the "receiving door" (12). Therefore, each door cannot be assigned to more than one truck (13).

$$\sum_h q_{mh} = 1 \quad \forall m \quad (11)$$

$$\sum_s y_{ms} = \sum_h q_{mh} \quad \forall m \quad (12)$$

$$\sum_m y_{ms} \leq 1 \quad \forall s \quad (13)$$

In the case of standard cross-dock, all loads unloaded are transported to the shipping doors in hub 1 (14) or hub 2 (15), according to the selected hub-door. Therefore, the loads handled within hub 1 (16) or hub 2 (17) must be equal to the number of loads that must be reloaded at the shipping doors of that hub (i.e., $\sum_j PD_{mj}$ for hub 1 and $\sum_p PD_{mp}$ for hub 2).

$$x_{mij} = y_{mi}\, PD_{mj} \quad \forall m, i, j \quad (14)$$

$$x_{mfp} = y_{mf}\, PD_{mp} \quad \forall m, f, p \quad (15)$$

$$\sum_i \sum_j x_{mij} = q_{mh} \sum_j PD_{mj} \quad \forall m,\ h=1 \quad (16)$$

$$\sum_f \sum_p x_{mfp} = q_{mh} \sum_p PD_{mp} \quad \forall m,\ h=2 \quad (17)$$

In addition, if there are some loads that are handle in the other hub, one of the three options must be selected for the handling. If there are loads handled in hub 1, equation 18 ensure that the truck choose only one of the three ways to manage the cross-dock (and similarly constrains 19 for hub 2).

$$\sum_i y_{mi} + \sum_i ynav_{mi} + \sum_f ytruck_{mf} = 1 \quad if\ \sum_j PD_{mj} > 0 \quad \forall m \quad (18)$$

$$\sum_f y_{mf} + \sum_f ynav_{mf} + \sum_i ytruck_{mi} = 1 \quad if\ \sum_p PD_{mp} > 0 \quad \forall m \quad (19)$$

Regardless of the choice to manage the cross-dock, all loads to be recharged in hub 1 (20) and in hub 2 (21) must be unloaded and transported to the shipping doors.

$$\sum_j x_{mij} + \sum_{h=1} \sum_f \sum_p xnav_{mhinfp} + \sum_{h=1} \sum_p xtruck_{mhidsp} = y_{m,i}\left(\sum_j PD_{mj} + \sum_p PD_{mp}\right) \quad \forall\, m, i, n \in n_1, ds \in ds_2 \quad (20)$$

$$\sum_p x_{mfp} + \sum_{h=2} \sum_i \sum_j nnav_{mhfnij} + \sum_{h=2} \sum_j ntruck_{mhfdsj} = y_{m,f}\left(\sum_j PD_{mj} + \sum_p PD_{mp}\right) \quad \forall\, m, f, n \in n_2, ds \in ds_1 \quad (21)$$

In addition, constraints (22) and (23) confirm that the flow of loads due to TDH or shuttle handling departs from the hub where the truck has performed the standard unloading. Therefore, only one of the two constraint variables (22) can be equal to 1 if the standard unloading hub is hub 1 and vice versa for hub 2 (see constraint 23).

$$\sum_f ynav_{mf} + \sum_i ytruck_{mi} \leq \sum_i y_{mi} \quad \forall m \qquad (22)$$

$$\sum_i ynav_{mi} + \sum_f ytruck_{mf} \leq \sum_f y_{mf} \quad \forall m \qquad (23)$$

In particular, in the case of shuttle-based handling, all loads moved by shuttle must be loaded at the same departure door, even if they come from different trucks. Similarly, they must be unloaded at the same receiving door, whether they come from hub 1 and are destined for hub 2 (24) or vice versa (25).

$$\sum_m \sum_f ynav_{mf} \ \leq \ 1 \qquad (24)$$

$$\sum_m \sum_i ynav_{mi} \ \leq \ 1 \qquad (25)$$

The number of loads managed by the shuttle cannot exceed its capacity W:

$$\sum_m \sum_{h\,=\,1} \sum_i \sum_{n \in n1} \sum_f \sum_p xnav_{mhinfp} \ + \sum_m \sum_{h\,=\,2} \sum_f \sum_{n \in n2} \sum_i \sum_j nnav_{mhfnij} \leq W \qquad (26)$$

Moreover, constraints (27) and (28) guarantee that the number of loads moved by shuttle in hub 1 (27) and hub 2 (28) and reloaded at the shipping doors is PD_{mp}.

$$\sum_{h=1} \sum_i xnav_{mhinfp} \ = ynav_{mf} \, PD_{mp} \qquad (27)$$

$$\forall \, m, f, p, n \in n_1$$

$$\sum_{h=2} \sum_f nnav_{mhfnij} \ = ynav_{mi} \, PD_{mj} \qquad (28)$$

$$\forall \, m, i, j, n \in n_2$$

Similarly, in the case of TDH, constraints (29) and (30) ensure that the number of loads moved to the charging doors is PD_{mp}.

$$\sum_{h=1} xtruck_{mhidsp} = ytruck_{mi} \, PD_{mp} \qquad (29)$$

$$\forall \, m, i, p, ds \in ds_2$$

$$\sum_{h=2} ntruck_{mhfdsj} = ytruck_{mf} \, PD_{mi} \qquad (30)$$

$$\forall \, m, f, j, ds \in ds_1$$

Finally, in case of TDH, constraints (31) and (32) ensure that the receiving door chosen in the second hub belong to the set of doors assigned to this option (ds_1 or ds_2) at each truck. This information is introduced by $RD_{m,ds}$.

$$\sum_{h=1} \sum_i \sum_p xtruck_{mhidsp} \, RD_{m,ds} \leq \sum_{h=2} \sum_p PD_{mp} \qquad (31)$$
$$\forall \text{ m}, \text{ds} \in ds_2$$

$$\sum_{h=2} \sum_f \sum_j ntruck_{mhfdsj} \ RD_{m,ds} \leq \ \sum_{h=1} \sum_j PD_{mj} \qquad (32)$$
$$\forall \text{ m}, \text{ds} \in ds_1$$

IV. CASE STUDY

The proposed MILP model is applied to an Italian company, leader in the express shipping of palletised products (called loads). This company collects and delivers loads through a network of logistic companies that capillary and geographically cover the whole country. Each company of this network (called *Affiliate*) provides one or more trucks for collecting products from several suppliers; then, these are sent to the nearest hub for unloading, sorting, and shipping upon customer order. For instance, a supplier x of Parma aims to ship a product x to a customer located in Terni. In this case, an Affiliate of Parma (or neighbouring) is responsible for collecting the products of several suppliers, including supplier x, and transporting them to the hub of Bologna. In this hub, the products are downloaded, sorted and reassigned to other Affiliates. In the specific case of product x, this is loaded on a truck of an Affiliate assigned to Terni, which will carry out the shipment. The company’s network, summarised in Fig. 4, comprises four hubs located one in Milano, two in Bologna and one in Napoli. All hubs are managed as cross-docks.

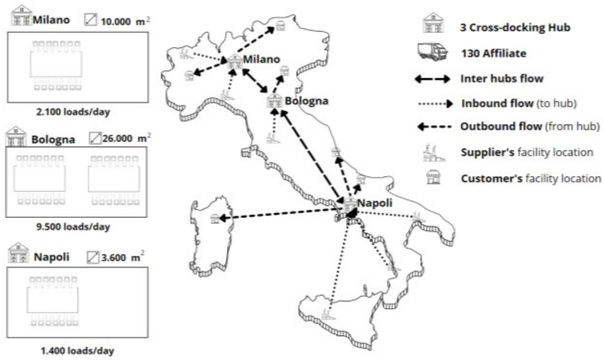

Fig. 4. Company’s Network

The proposed model focuses on the cross-dock handling activities carried out in the two twin hubs of Bologna. The presence of the second hub supports the sorting and shipping activities, increasing the total storage capacity. This solution was born because the first hub could not be expanded due to space limitations. The two hubs count 238 doors for inbound and outbound trucks. Therefore, given an incoming truck, the previously defined “hub-door assignment problem” must be solved. In addition, inter-hubs loads flow is managed by a dedicated shuttle or by TDH.

The application of the proposed optimisation model to this case study aims to compare the optimal solution with the actual hub-door assignment company policy.

Currently, the company policy requires that each truck arriving at the hubs of Bologna chooses hub 1 for unloading its loads if more than ten loads are destined for hub 1. Otherwise, the truck carries out the unloading in hub 2 if it transports less than ten loads destined for hub 1 and more than 10 for hub 2. Once the unloading hub has been identified, the receiving door is selected at the discretion of the hub manager. The existing tool that supports the manager's decision-making process is a Microsoft Excel document. It reports the following details for each truck arriving at the hub: the arrival date and time, the identification name of the arrival Affiliate, the number of loads and the shipping doors of each load). Finally, if there are some loads destined for the other hub, the handling by shuttle is preferred if there are less than ten loads to move to the other hub. Otherwise, the TDH option is performed.

The mathematical programming language AMPL and the solver Gurobi support the application of the proposed model to this case study. This approach provides the splitting of the optimisation problem into two files:

1. *.mod* file contains the parameters, variables, object function and constraints.
2. *.dat* file contains the data entry of the model.

Consequently, different solutions are provided according to the values reported in the .dat file. In this case study, the parameters are collected on-field and summarised in Table 3. Time and distance matrices between all door pairs are not reported.

TABLE III
CASE STUDY: SET AND PARAMETERS

Set, Parameter	Description
$h = 1,2$	*number of Hub*
$m = 1, \dots 4$	*number of inbound trucks*
$i = j = 1, \dots 118$	*receiving/shipping doors in Hub 1*
$f = p = 119, \dots 238$	*receiving/shipping doors in Hub 2*
$n_1 = 32$	*departure door for the shuttle in Hub 1*
$n_2 = 150$	*departure door for the shuttle in Hub 2*
$ds_1 = 48, 54$	*receiving doors for the TDH in hub 1*
$ds_2 = 173, 198$	*receiving doors for the TDH in hub 2*
$T^* = 12670$ sec	*handling time for moving loads to the departure door of the shuttle*
$T^*_{mov} = 6576$ sec	*TDH time*
$T^*_{op} = 1800$ sec	*time to arrange the shuttle*
$T\_SH^* = 100$ sec	*time for loading/unloading the load on/off the shuttle*
$T\_TDH^* = 80$ sec	*time for loading/unloading the load on/off the truck in the TDH*
$V^* = 2.77$ m/s	*vehicle speed*
$A^* = 0.4$ m^2/s	*vehicle acceleration time with load*
$A_e^* = 0.6$ m^2/s	*vehicle acceleration time without load*
$W = 70$ load/trip	*shuttle capacity*

* values measured on the field or taken from the forklifts datasheet

The proposed MILP model is applied considering an increasing number of inbound trucks. The purpose is to compare the solution provided by the MILP model with the one supported by the company policy and measure whether the hub-door assignment is affected by multiple trucks' simultaneous unloading.

Firstly, the proposed MILP model is applied to a single truck configuration, named T1 and illustrated in Fig. 4, to assess the first objective mentioned above. The features related to the number of loads transported by truck T1 and the charging door of each load are shown in Figure 5. Table 4 presents the comparison between the two solutions by explaining the hub-door assignment (*Hub*, *Door*), the option selected to manage the inter-hubs loads flow (Shuttle – *SH* or Truck Double Handling– *TDH*) and the total time required to unload, sort and ship all the loads (*Time*).

TABLE IV
CASE STUDY: SINGLE TRUCK

	MILP model	Company policy
Hub	*T1: Hub 2*	*T1: Hub 1*
Door	*T1: Door 137*	*T1: Door 30*
Option	*T1: SH*	*T1: TDH*
Time	*142.43 min*	*266.03 min*

The optimal solution proposed by the MILP model differs from the company policy one and saves about 46% of the time. This percentage is specific to this single truck. Nevertheless, it is enough to claim that the optimal solution improves handling activities compared to the current solution.

Moreover, the MILP model has been applied to up to 4 trucks (called T1, T2, T3, T4) to reach the second objective and verify if the solution changes with the increased number of trucks. Figure 5 illustrates the features of each load transported by the four trucks.

Hub 1

TRUCK	Door	# Loads
T1	8	1
	24	1
	54	1
	73	1
	92	4
	96	2
T2	15	1
	20	2
	24	2
	50	1
	54	1
	67	1
	92	4
	96	9
	97	4
T3	18	2
	27	1
	46	3
	48	2
	51	1
	57	1
	77	1
	82	1

TRUCK	Door	# Loads
T3	85	1
	90	1
	103	2
	107	1
	113	1
T4	6	2
	9	2
	12	1
	17	1
	18	2
	22	2
	24	3
	27	5
	43	1
	57	1
	81	1
	92	4
	97	1
	103	2
	106	1
	111	2
	114	2

Hub 2

TRUCK	Door	# Loads
T1	137	8
	198	1
T2	124	1
	180	1
	181	1
	184	1
	198	2
	200	4
	213	2
T3	120	4
	129	1
	131	1
	137	1
	153	1
	155	1
	164	1
	176	1
	178	1
	179	4
	180	1
	182	1
	185	3
	190	1
	194	1

TRUCK	Door	# Loads
T3	203	2
	205	4
	214	1
	219	1
	235	1
T4	121	1
	128	1
	131	6
	132	1
	142	1
	155	3
	158	1
	159	3
	161	1
	163	1
	182	1
	186	2
	194	2
	200	2
	203	1
	205	1
	215	1
	221	2
	230	2

Truck T1 Charging Door: 48 - Hub 1
Truck T2 Charging Door: 54 - Hub 1
Truck T3 Charging Door: 200 - Hub 2
Truck T4 Charging Door: 173 - Hub 2

Fig. 5. Features of the loads transported by the four trucks

Table 5 presents the solutions generated by the MILP model considering an incremental number of trucks (from one to four).

TABLE V
CASE STUDY: MULTI-TRUCK

	One Truck	Two Trucks	Three Trucks	Four Trucks
Hub	*T1: Hub 2*	*T1: Hub2*	*T1: Hub2*	*T1: Hub2*
		T2: Hub 2	*T2: Hub 2*	*T2: Hub 2*
			T3: Hub 1	*T3: Hub 1*
				T4: Hub 1
Door	*T1: 137*	*T1: 137*	*T1: 137*	*T1: 137*
		T2: 200	*T2: 199*	*T2: 199*
			T3: 46	*T3: 46*
				T4: 27
Option	*T1: TDH*	*T1: TDH*	*T1: TDH*	*T1: TDH*
		T2: TDH	*T2: TDH*	*T2: TDH*
			T3: TDH	*T3: TDH*
				T4: TDH
Time [min]	*142.43*	*343.95*	*569.20*	*834.90*

Data reported in Table 5 illustrates how the hub-door assignment changes if the model considers multi-truck unloading simultaneously. For instance, considering trucks T1-T2-T3 the unloading door of T2 switches from 200 to 199. Indeed, door 200 will be used by T3 for TDH and is no longer available for standard unloading. Therefore, the proposed model minimises the material handling time of multi-truck in a multi-door cross-dock considering the interactions between trucks and managing the inter-hubs flows.

V. CONCLUSION

Optimising the material handling activity is of great importance in reducing shipping time. Networks built on hubs managed as cross-dock are competitive only if they guarantee short shipping times.

This study introduces an original mixed-integer linear programming (MILP) to solve the dock-assignment problem in a multi-door cross-docking system, where multiple trucks can simultaneously unload and reload palletised until-loads. This system is made of two twin hubs. Having two nearby hubs is increasingly widespread in modern networks, and rarely covered by literature. The proposed model minimises the material handling time required to manage the intra- and inter-hubs loads flows. Finally, through the application to a real case study, the proposed solution is compared with the actual company's policy. The comparison illustrates the relevant saving that the MILP model allows reaching considering only one truck.

The potential of the model lies in its scalability. Therefore, the temporal savings obtained from the management of a single truck can be extended to more complex situations, that they regard the unloading of more means in contemporary.

Further research is expected on the integration with network management models, aimed at optimising performance parameters such as transport costs and truck utilization.

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