EOQ: a simulation approach for perishable products

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Abstract: It is well known that the inventory cost represents a non-negligible quota of the total logistics cost and for this reason, nowadays there are several models and approaches available to optimize its management. Minimizing the inventory cost, however, is challenging, because of the huge number of factors involved, some of which are stochastic and difficult to control. Because of this complexity, there are no general approaches suitable to be applied in any context or scenario. The problem is exacerbated when handling perishable products, that cover a huge part of products such as food or pharmaceutical items.

In this paper, we revisit the classical Economic Order Quantity (EOQ) inventory model and develop a simulation approach to determine the control policies for perishable products. The goal of the model is to reduce the total inventory management cost by determining the best combination of order quantity (OQ) and order point (OP). To be more precise, using a simulation approach, an optimal trade-off between the inventory cost, order cost, stock out cost, and disposal cost is determined. The model takes into account a uniformly distributed stochastic demand and assumes a fixed replenishment lead time; it also admits backorders when the available stock is not sufficient to meet the demand and adopts a First-In-First-Out policy (FIFO) to satisfy customer orders. Finally, the impact of product shelf-life and procurement lead time on system behavior is evaluated, by defining a proper simulative campaign and performing a sensitivity analysis on the results obtained. This study aims to help the decision makers in predicting how the perishability of the product and the choice of the supplier would impact on the total management cost, improving their understanding of the system behavior and increasing their control over its variables.

Keywords: perishable products, simulation approach, EOQ, inventory management, shelf-life

I. INTRODUCTION AND LITERATURE REVIEW

The term *perishable* is used to describe an item, usually food or pharmaceutical, that spoils within a relatively short amount of time. Moreover, perishability can also define a branch of products that with the passing of time become obsolete, as in the case of electronic equipment. In light of the intrinsic criticality of these items, it is usually necessary to implement *ad hoc* policies for their storage management, aiming to minimize storage costs and reduce the waste of products.

[1] has estimated that the total cost of unsalable products, including items that have reached the expiry date in drugstores and supermarkets accounted for 2.05 billion dollars in 2005. Consequently, non-optimized management of the stocks of perishable products will lead to high management costs as well as food wastage, which is in contrast to the sustainable management and development of the enterprise and society in general. This last point is also one of the sustainable development goals (SDGs) set by the United Nations agenda [2].

The main models for inventory management, e.g., Economic Order Interval (EOI), Economic Order Quantity (EOQ), and (S,s) policies, assume infinite shelf-life of stored items. The management of perishable products is however more challenging compared to long-lasting products [3], and, consequently, widely investigated in the literature. In particular, [4] developed a numerical method with First-In-First-Out (FIFO) or Last-In-First-Out (LIFO) issuing policy, non-stationary demand and service level constraints, to determine dynamic order quantities for perishable products with limited (fixed) shelf-life. Other authors have developed a stochastic mathematical model. [5], in particular, modelled the demand as a stochastic variable with normal distribution. The demand for perishable products is also sensitive to changes in the item price; accordingly, the recent work by [6] has developed a mathematical model in which the demand is dependent on stocks. As the demand can be sensitive to price, a vendor may need to backlog demand to avoid costs due to product deterioration. In this respect, [7] proposed a model in which perishable products have a price-dependent demand, and partial back-ordering and sale losses are admitted. The model is solved analytically to obtain the optimal price and size of the replenishment.

Other reviews of these models focusing expressly on inventory management for perishable items have been developed and published in the literature, as the work by [8] - [11].

In recent years the simulation approach has been increasingly adopted for the optimization of inventory management systems [12] - [14]. Simulation, and particularly discrete-event simulation, unlike analytic models, does not require strong assumptions, thus resulting more suitable for real case studies.

This paper presents a numerical solution for the management of a warehouse for perishable products using a discrete-event simulation model, developed with

MS Excel™. The management is performed according to an EOQ policy, which is also called "continuous review" because the inventory status needs to be tracked continuously [15].

The continuous review policy was chosen in light of the very limited literature about the usage of simulative approaches for the optimization of EOQ-based inventory systems for decaying items. This paper, therefore, aims to contribute to scientific knowledge by presenting a multi-period simulation model for perishable products.

The products considered in this study are ordered from a supplier and, after a fixed and deterministic procurement lead time, they are delivered to the warehouse and used to satisfy a uniformly distributed customer demand according to a FIFO policy. The products have a fixed and deterministic shelf-life, that decreases daily, starting from the moment the order is shipped by the supplier. The commercial value of the items stored is assumed to be constant until the expiration date, when the products lose their value altogether and must be disposed of.

The goal of this study is to optimize inventory management, by determining the best combination of EOQ and order point (OP) that allow for minimizing the total system cost. Finally, the analysis aims to evaluate the impact of product shelf-life and procurement lead time on the optimal operating leverage values, by defining a proper simulative campaign and performing a sensitivity analysis on the results obtained.

II. NOMENCLATURE

The nomenclature adopted is presented in Table 1.

TABLE 1
NOMENCLATURE OF THE MODEL EVALUATED

Symbol	Description	Unit
N	Number of days	days
i	i-th day	-
LT	Procurement lead time	days
SL	Perishable product shelf-life	days
d_M	Mean demand value	units
d_R	Half demand variation range	units
OP	Order Point	units
OQ	Order Quantity	units
OHi	On-hand inventory at day i	units
New	Products received with the latest order	units
Old	Products received with the previous order	units
$Q_{exp,i}$	Amount of product expired at day i	units
OOS_i	Amount of out-of-stock at day i	units
C_{oi}	Order issuing cost	€/order
c_{so}	Unitary stock-out cost	€/unit
$C_{so,i}$	Stock-out cost at day i	€/day
c_{inv}	Unitary inventory holding cost	€/unit
$C_{inv,i}$	Inventory holding cost at day i	€/day
c_{disp}	Unitary disposal cost	€/unit
$C_{disp,i}$	Disposal cost at day i	€/day
$C_{tot,i}$	Total inventory management cost at day i	€/day
$C_{tot,min}$	Minimum daily total cost	€/day
OP_{opt}	OP that minimizes the total cost	units
EOQ	Order size that minimizes the total cost	units

III. MODEL OVERVIEW AND ASSUMPTIONS

An EOQ-based, continuous review, inventory control model was developed using the nomenclature detailed above. A discrete events simulation model was generated in MS Excel™, to simulate a Distribution Center (*DC*) that receives the ordered lots of products from a supplier, and then uses them to fulfill the daily demand of several customers (retail stores). A single-product scenario was evaluated considering the perishability of the items, that are assumed to have a predefined deterministic shelf-life. The perishable products maintain their commercial value unaltered until the end of their shelf-life; at that point, they instantly lose all their value and must be disposed of ($Q_{exp,i}$), generating a disposal cost ($C_{disp,i}$), calculated with eq. 1.

$$(1) \quad C_{disp,i} = c_{disp} \cdot Q_{exp,i}$$

The stock-on-hand (OH_i) is continuously reviewed, according to the EOQ policy, and a constant quantity of items (*OQ*) is ordered whenever the stock level falls below a predefined reorder point (*OP*). The order issuing cost (C_{oi}) is fixed and independent of the ordered quantity, and it is determined based on the administrative costs associated with order processing, transport, receiving, and unloading. Once an order is issued, the products are received and become available in the physical stock after a procurement lead time (*LT*), which is again fixed and deterministic.

The products of the same order are assumed to have the same shelf-life, which starts decreasing after the order issuing, and scores *SL-LT* when the ordered lots are delivered to the warehouse. The cost of keeping the products in the warehouse ($C_{inv,i}$) is determined as a function of the quantity stored (eq. 2).

$$(2) \quad C_{inv,i} = c_{inv} \cdot OH_i$$

The customer's daily demand is assumed to be stochastic, with uniform distribution and known mean and standard deviation, and it is processed according to a rigorous First-In-First-Out (*FIFO*) policy. When the stock-on-hand is not sufficient to satisfy the demand, an out-of-stock (OOS_i) situation is generated. Shortages are allowed and fully back-ordered. The delayed fulfillment of customers' orders generates additional costs, which are assumed to be embodied in the stock-out cost (eq. 3).

$$(3) \quad C_{so,i} = c_{so} \cdot OOS_i$$

Since back-ordering is allowed, an additional condition is included in the model. When an order is delivered, if the stock-on-hand left in the warehouse after fulfilling the delayed orders is less than OP, a new order is immediately issued.

Considering the assumptions discussed above, four main cost items can be identified: inventory holding cost, stock-out cost, order-issuing cost, and disposal cost. The total daily cost is calculated as the sum of all costs incurred by the company each day (eq. 4):

$$(4) \quad C_{tot,i} = C_{oi} + C_{inv,i} + C_{so,i} + C_{disp,i}$$

The average total cost in the period considered is calculated by dividing the sum of each daily C_{tot} by the number of days in the period. To obtain reliable results from a statistical point of view, a period of 50,000 days was simulated. By further increasing the number of periods analyzed, no changes in the results obtained were observed. The following equation is used for the computation (eq. 5):

$$(5) \quad C_{tot} = \frac{\sum_{i=1}^{N} C_{tot,i}}{N}$$

The optimal management condition is defined as the combination of *EOQ* and OP_{opt} that allows the company to minimize C_{tot} ($C_{tot,min}$). A graphical representation of the model described is reported in Fig. 1.

The level of the stock on-hand during the period evaluated is represented in Fig. 2. When an order is issued, the quantity ordered is immediately added to the theoretical stock level, and it is included in the physical stock after LT days, when the items ordered are delivered to the warehouse. According to the FIFO policy, products delivered earlier, referred to as "Old" in this paper, are the first to be used to satisfy the customer's demand, therefore leaving the level of "New" products unaltered. When the available "Old" products are not enough to meet the demand, "New" products can be sold, and their stock level begins to decrease. A sudden decrease in the stock on-hand can be observed when the products reach the end of their lifetime and must be disposed of.

IV. NUMERICAL SIMULATION

A period of 50,000 days was simulated using MS Excel™ and the inventory model discussed above. The simulations were carried out to assess the impact of the product shelf-life and procurement lead time on optimal management conditions.

Three case studies, with different *LT* values, were modeled and, each time, the optimal management conditions, defined as the combination of the operating leverages (*EOQ* and OP_{opt}) that allows the company to minimize the total inventory management cost, were evaluated. Procurement lead times of 3, 5, and 7 days were analyzed. Product SL values between $2 \cdot LT$ and 20 days were considered, increasing the value by one for each simulation. As can be seen from Fig. 3, for shelf-life values less than $2 \cdot LT$ (SL=10 days and SL=12 days), the products received LT days after the order issuing expire before the next stock replenishment and have to be disposed of, so they cannot be used to satisfy the customer's demand. Disposal and out-of-stock, therefore, occur regularly at each reordering cycle. For shelf-life values at least equal to $2 \cdot LT$ (SL=14 days and SL=16 days), it can be appreciated that out-of-stock situations are not so regular and, as a consequence, an acceptable level of customer service is achieved. This approach allowed performing sensitivity analysis and evaluating the impact of SL and LT on the system behaviour. To this end, the values of the input parameters are reported in Table 2.

TABLE 2
VALUES OF INPUT FACTORS USED FOR THE FIRST SET OF SIMULATIONS

Factor	Value	Unit
c_{inv}	0.022	€/unit
c_{so}	0.44	€/unit
c_{oi}	1200	€
c_{disp}	0.18	€/unit
d_M	2500	units
d_R	800	units
SL	6 - 20	days
LT	3 – 5 – 7	days

V. RESULTS AND DISCUSSION

The results of the simulation campaign relating to the three case studies evaluated are presented in Table 3.

To graphically show the impact of *SL* and *LT* on the system behaviour, graphs of the calculated values of $C_{tot,min}$, *EOQ* and OP_{opt} are presented in Fig. 4-6 for increasing values of *SL*. Each graph contains three plots referring to the 3 values of *LT* simulated.

In Table 3 it can be appreciated that, as the procurement lead time increases, *EOQ* tends to approach OP_{opt} value. In this configuration, while waiting for the ordered quantity, the demand is fulfilled using the stocks corresponding to OP_{opt}. Therefore, once the ordered quantity is received, at the end of *LT*, it is used to restore a stock level close to OP_{opt}. Another order has then to be issued immediately or, at the latest, the next day. The management policy then becomes a periodic review policy with the reorder period equal to *LT*.

Fig. 4 highlights that $C_{tot,min}$ increases as *LT* increases and decreases as *SL* increases.

Fig. 5 shows that for low *SL* values (slightly higher than 2**LT*), *EOQ* increases as the shelf-life increases. It also points out that, as *LT* increases, the dependence of *EOQ* on shelf-life, and therefore on the perishable nature of the products, progressively decreases.

Fig. 6 highlights that OP_{opt} is largely influenced by *LT* and negligibly influenced by *SL*. In particular, it increases with increasing *LT*, while it is almost independent on the shelf-life value.

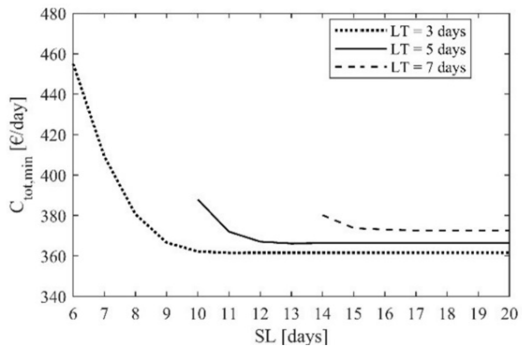

Fig. 1 Trends of minimum system cost at different SL, evaluated at three procurement lead time periods

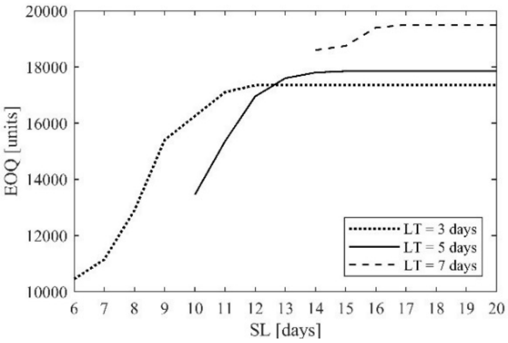

Fig. 2 Trends of EOQ at different product SL

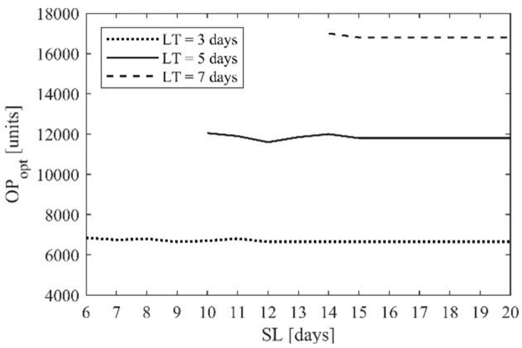

Fig. 3 Trends of OP_{opt} at different *SL* periods

This can be explained by the fact that OP_{opt}, regardless of the shelf-life value, corresponds to that stock level which is sufficient to satisfy the demand observed in the period between the order issuing and the receipt of the items (period corresponding to *LT*). In light of this, it can be concluded that, reasonably, the most significant factors in the determination of OP_{opt}, are *LT* and d_M.

Finally, in all the figures just described, it can be seen that, in each configuration, above a certain shelf-life value, both the operating leverages that allow for minimizing the total inventory management cost (OP_{opt} and *EOQ*) and the minimum daily total system cost ($C_{tot,min}$), are no longer influenced by the shelf-life. It can therefore be concluded that, above these shelf-life values, the perishable nature of the products can be considered negligible in defining the warehouse reordering parameters and in estimating the inventory management costs.

VI. CONCLUSIONS

We have developed a discrete event multi-period simulation model for inventory management in a distribution center according to a continuous review replenishment policy. The model reproduces a single product scenario, in conditions of deterministic lead time, deterministic shelf-life, and stochastic demand with uniform distribution. Back-orders were admitted, and it was assumed that the products maintained their commercial value unchanged until the end of their useful life.

A sensitivity analysis was performed to evaluate the impact of both shelf-life (*SL*) and procurement lead time (*LT*) on the choice of the best combination of the operating leverages (OP_{opt} and E*OQ*) and on the total management cost ($C_{tot,min}$).

The results showed that *SL* significantly impacts both $C_{tot,min}$ and *EOQ*. In particular, $C_{tot,min}$ decreases and EOQ increases while *SL* increases. Moreover, as *SL* increases in value, its impact, both on $C_{tot,min}$ and *EOQ*, progressively decreases, until it becomes almost irrelevant above a given shelf-life value. In these conditions, the perishable nature of the products can be neglected and, from a warehouse management point of view, an infinite shelf-life could be considered. OP_{opt}, on the other hand, does not result to be significantly influenced by *SL*, whatever its specific value.

As regards the effects of the procurement lead time, the most significant impact was observed for $C_{tot,min}$ and OP_{opt}. In particular, both parameters increase as LT increases.

In future studies it will be interesting to compare continuous review with periodic review and (S,s) management policies, to evaluate the performance of the different inventory management policies in perishable conditions. Furthermore, the assessment of the impact that the assumption of uniform demand distribution has on inventory management, is an interesting aspect that deserves further investigation. Moreover, additional studies could be performed in conditions of stochastic lead time and without the possibility of back-order. Other assessments could be carried out to include constraints on order size or warehouse capacity; a more realistic analysis could also be performed by evaluating a multi-product distribution center.

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Appendix A. IMAGES AND TABLES ON 2 COLUMNS

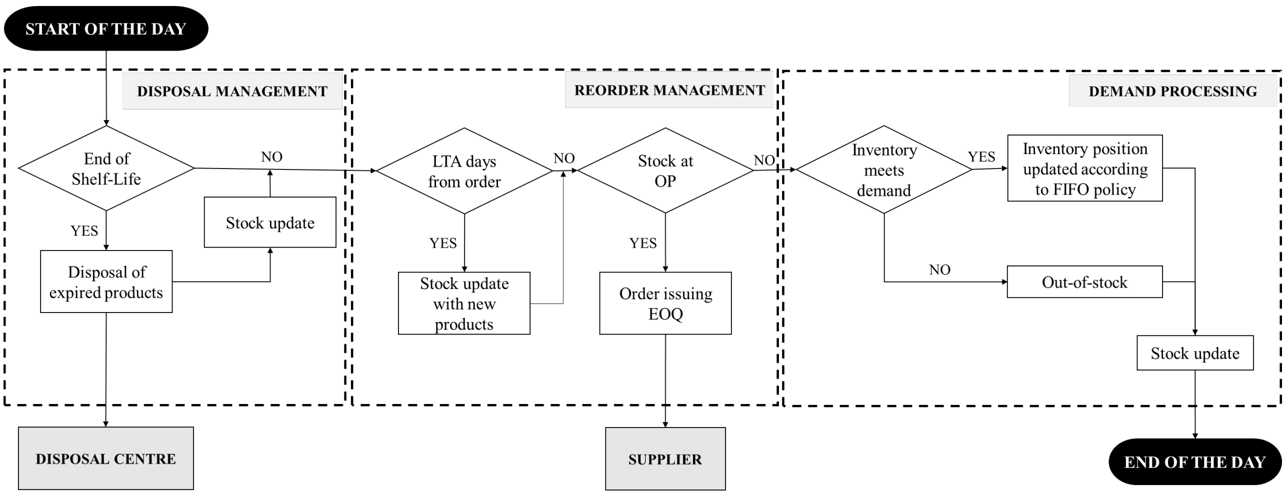

Fig. 4 Flowchart of the main warehouse management operations performed each day

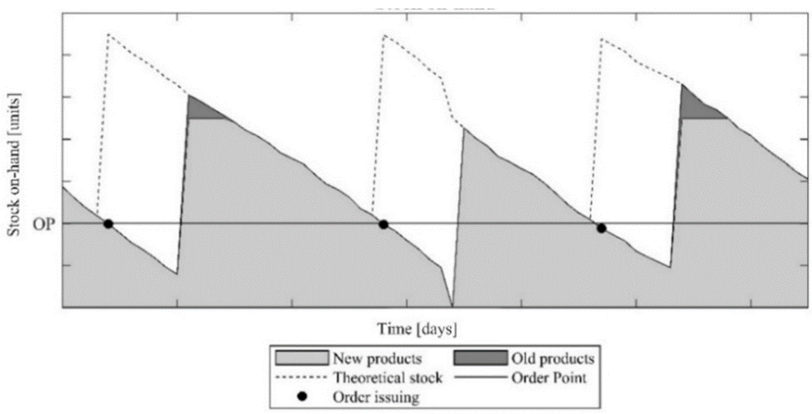

Fig. 5 Inventory trend of a warehouse managed according to a FIFO policy and an EOQ-OP reorder model

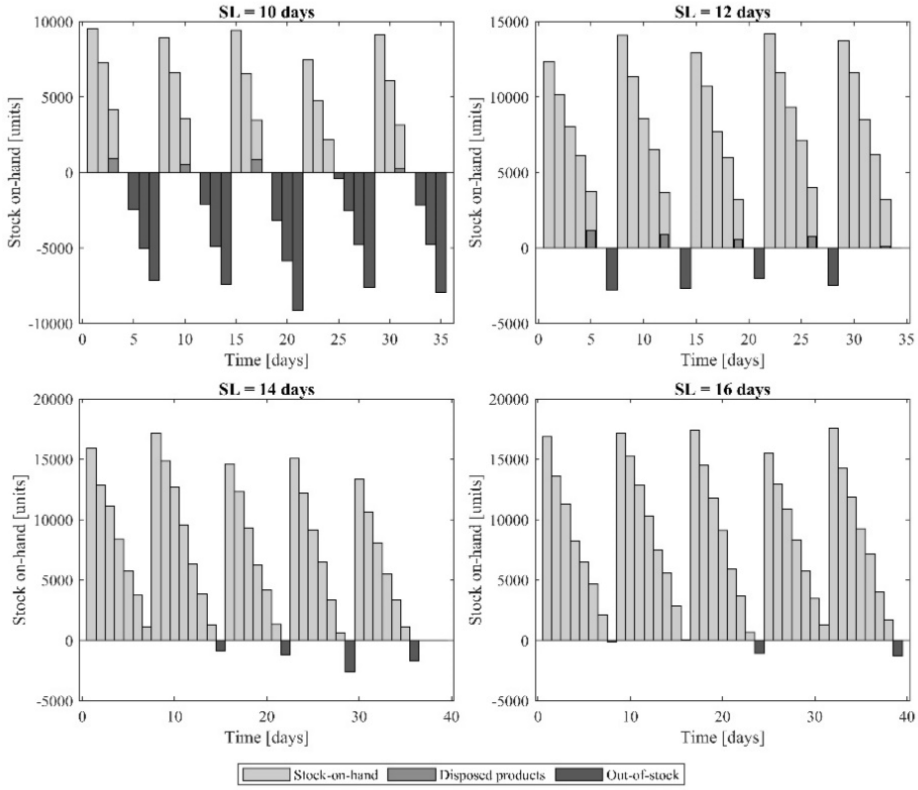

Fig. 6 Stock-on-hand trend for different *SL* values and *LT*=7 days

TABLE 3
RESULTS OF THE SIMULATION CAMPAIGN

	LT = 3 days			LT = 5 days			LT = 7 days		
SL [days]	$C_{tot,min}$ [€/day]	OP_{opt} [units]	**EOQ** [units]	$C_{tot,min}$ [€/day]	OP_{opt} [units]	**EOQ** [units]	$C_{tot,min}$ [€/day]	OP_{opt} [units]	**EOQ** [units]
6	455.07	6850	10450	-	-	-	-	-	-
7	409.26	6750	11150	-	-	-	-	-	-
8	380.66	6800	12900	-	-	-	-	-	-
9	366.63	6650	15400	-	-	-	-	-	-
10	362.29	6700	16250	387.90	12050	13450	-	-	-
11	361.55	6810	17100	372.06	11900	15350	-	-	-
12	361.58	6650	17350	367.10	11600	16950	-	-	-
13	361.58	6650	17350	366.21	11850	17600	-	-	-
14	361.58	6650	17350	366.38	12000	17800	380.26	17000	18600
15	361.58	6650	17350	366.45	11800	17850	373.82	16800	18750
16	361.58	6650	17350	366.45	11800	17850	373.07	16800	19400
17	361.58	6650	17350	366.45	11800	17850	372.66	16800	19500
18	361.58	6650	17350	366.45	11800	17850	372.66	16800	19500
19	361.58	6650	17350	366.45	11800	17850	372.66	16800	19500
20	361.58	6650	17350	366.45	11800	17850	372.66	16800	19500